Expanding Role of Potential Theory in Supersonic Aircraft Design

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The application of linearized theory to the external aerodynamic design and analysis problem at supersonic speeds is discussed. Particular emphasis is placed on the use of a far field viewpoint since this concept makes it possible to treat thickness drag from a total system standpoint. For the lifting case, such an approach results in a decomposition of aircraft resistance into shock wave and vortex components. This decoupling allows an identification of the importance of such losses relative to one another and permits considerable latitude in modifications to reduce the drag in one mode without adversely interacting with the other. Representative comparisons of prediction with measurements are presented to illustrate the accuracy and limitations of the various procedures. Certain unresolved difficulties which arise in the application of potential theory to the aircraft design problem will be pointed out to identify areas requiring further development.

Nomenclature

A = cross-sectional area
b = wake width or wing span
c = local chord
C_D = drag coefficient $- D/qS_W$
C_d = sectional drag coefficient
C_L = lift coefficient $- L/qS_W$
C_l = sectional lift coefficient
C_p = static pressure coefficient
D = drag
F_θ = component of force in θ direction
$= q \int_0^\epsilon \int_C c_p(\epsilon,\theta)\,(dy \sin\theta + dz \cos\theta) d\epsilon$
L = lift
M = freestream Mach number
p = freestream static pressure
P_T = total pressure
q = freestream dynamic pressure
S = oblique projected cross-sectional area
S_W = wing area
T = thrust
u,v,w = x,y,z perturbation velocities
U = freestream velocity
x,r,θ = cylindrical coordinate system
x,y,z = wind axis cartesian coordinate system
α = angle of attack
β = $[M^2 - 1]^{1/2}$
γ = ratio of specific heats
$\epsilon,\epsilon_1,\epsilon_2$ = dummy longitudinal variables
ρ = freestream density
ϕ = velocity potential

Subscripts

J = jet
L = due to lift
T = throat
LE = leading edge
V = vortex
W = wave
x,y,z,r = $\partial/\partial x, \partial/\partial y, \partial/\partial z, \partial/\partial r$
xx,yy,zz = $\partial^2/\partial x^2, \partial^2/\partial y^2, \partial^2/\partial z^2$
VIS = viscous
NOZ = nozzle
π = based on maximum cross-sectional area

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I. Introduction

UTILIZATION of the linearized form of potential theory for supersonic aircraft aerodynamic design and evaluation has increased markedly in recent years.

The pertinent equations of motion are well known and appear in standard text books on compressible gasdynamics. In addition, applications of the theory to slender bodies of revolution, thin wings, or combinations of a thin wing centrally mounted on a cylindrical body have been published for some time. Unfortunately these analyses were often restricted to an idealized class of geometry to obtain solutions, so the results have received relatively limited use for aircraft design purposes.

The basic objective of more recent developments is the numerical application of the theory to nonsimple multicomponent systems through use of high-speed digital computers. An example of this type of development, representative comparisons of the results with experimental data, and the design concepts which result are the subject of this paper. As a part of this presentation, a brief review of the application of linearized theory to aircraft design will be given to establish continuity between various current and past developments.

II. Technical Discussion

The basic technique which will be utilized is the linearized form of small perturbation theory for steady supersonic frictionless flow[1]

$$(M^2 - 1)\phi_{xx} - \phi_{yy} - \phi_{zz} = 0 \qquad (1)$$

The streamwise slope of the disturbing object must therefore be everywhere sufficiently small that the small perturbation hypothesis adequately describes the major domain of the flow. Further, conditions resulting in flows having significant regions which do not satisfy the linearization requirements $M^2 - 1 \gg M^2(\gamma + 1)u/U$ and $M^2(\gamma + 1)u/U, M^2v/U, M^2w/U \ll 1$ are excluded.

In general, the previous restrictions do not rule out neighborhoods (such as stagnation points, etc.) where the perturbation velocities are large provided such regions are limited in extent and number; it is usually found that the linearized analysis is still representative of the flow in the large.

Although the approximations to the exact potential equation of motion are many, the linearized expression has received widespread use; first, because it is the only adequate theory with sufficient generality to treat systems with rela-

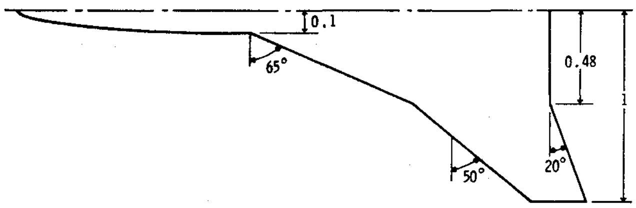

Fig. 1a Comparison of lifting-surface theory and measurement—test configuration.

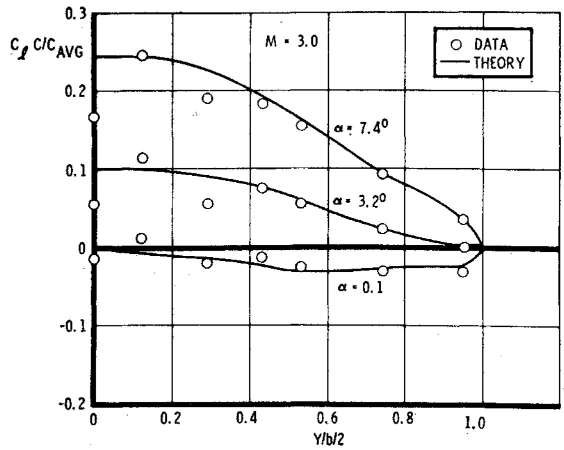

Fig. 1b Comparison of lifting-surface theory and measurement—spanwise lift.

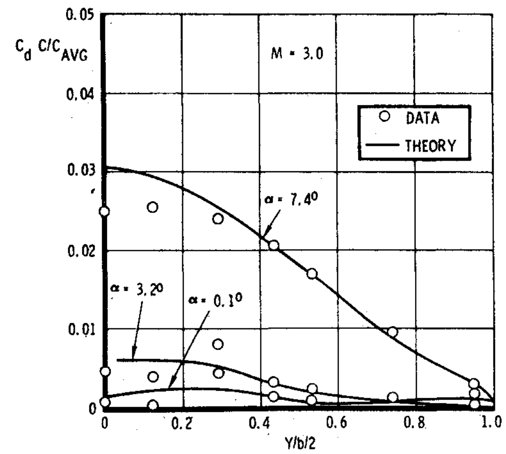

Fig. 1c Comparison of lifting-surface theory and measurement—spanwise drag.

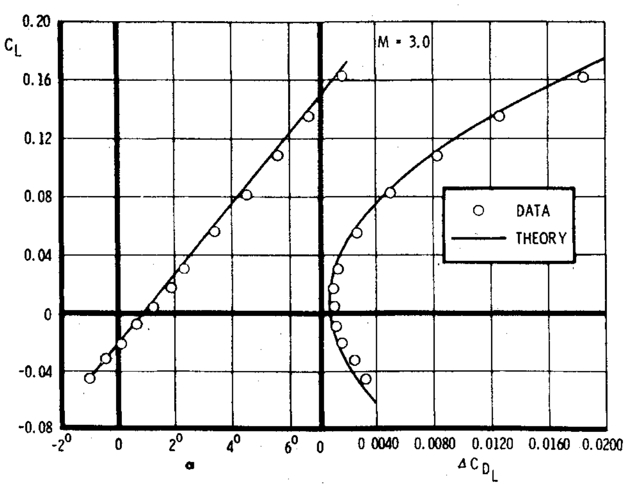

Fig. 1d Comparison of lifting-surface theory and measurement—force characteristics.

tively complex boundaries, and second, because it is reasoned that aircraft producing a large disturbance in the external flow would be inherently inefficient. As a consequence, the theory would be expected to work best for those cases where high aerodynamic efficiency is a primary design goal.

The earliest application of Eq. (1) was in the estimation of the drag due to volume of a slender projectile.[2] This problem was in fact responsible for the original linearization of the equation of motion. This was followed by analytical analyses[3–7] to determine body meridal contour having minimum drag subject to various constraints such as fixed volume, length, base area, etc. More recently, the numerical determination of the shape and resistance of optimum bodies subject to a variety of multiple constraints (including the previous work as special cases) has been reported.[8] The application of Eq. (1) to the lifting nonaxisymmetric body has been reported by Barnett.[9]

A second class of problems to which linearized theory was applied fairly early was in the determination of the relative volume[10–12] and lifting[11–15] efficiency of certain restricted classes of thin uncambered wings. This effort was followed by analyses[16–19] concerned with obtaining improvements in lifting efficiency for a specified planform by longitudinally warping the mean surface out of a plane. More recently, numerical codes for evaluating the lifting characteristics of nonsimple planforms with twist and camber[20,21] and camber surface for least drag[22] have been developed. An example of linearized prediction and measurements for the former case based on source sheet formulation to Eq. (1) and the Evvard-Etkin solution procedure is presented in Fig. 1. Comparisons using a vortex formulation have been reported by Middleton and Carlson.[23]

Initial studies[14] of a thin wing-slender body combination simply added the theoretical lift and drag characteristics for the isolated wing and body together. Subsequently, refined analysis taking into consideration the mutual interference between the components was published.[24–27] Two different lines of attack were followed in this work. The first utilized solutions for the disturbance velocity at a great distance (far field viewpoint) from the wing body. A discussion of this concept will be deferred until later as it will form the primary emphasis for the remainder of the paper. The second procedure utilized the standard approach of obtaining solutions for the disturbance field at the surface (near field viewpoint) of the wing body. This is the technique which was used exclusively in the previous theoretical analysis of slender bodies and thin wings. Although solutions[26,27] based on this latter approach succeeded in establishing a more comprehensive theoretical analysis of the wing-body problem, they were limited to centrally mounted panels on a cylindrical body. In addition, the wing planform had numerous restrictions for the lifting case. As a consequence, these results were not usable to any great extent for aircraft design purposes. This situation remained essentially unchanged for 10 years until the recent development reported by Woodward.[28] This effort is particularly noteworthy as it represents the first systematic near-field application of linearized theory to the wing-body problem that treats both thickness and lift in sufficient generality to be useful for design. An assessment of the accuracy and limitations of this technique has been reported by Carmichael.[29]

Far Field Viewpoint

The theoretical technique and design concepts which are deduced by using distant asymptotic solutions to Eq. (1) will now be considered.

The basic approach which is employed relates the resistance of a slender lifting object to the momentum flux through a control volume enclosing the object in accordance with Newton's second law. Choice of a circular cylinder (Fig. 2) whose axis is alined with the freestream velocity vector results in the following expression[30,31] for total pressure drag:

$$D = -\rho \iint r\phi_x\phi_r dx d\theta + \frac{\rho}{2} \iint (\beta^2\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2) dy dz \quad (2)$$

The first term represents the wave resistance and results from the momentum per unit time carried through the sides of the cylinder by the standing pressure waves created by the object. The second term represents the vortex drag which arises from the momentum flux transported downstream by the trailing vortices.

A standard technique for estimating the nonviscous drag of a smooth slender object in a steady supersonic flow is to simulate the physical system by a distribution of elementary solutions (singularities) to the linearized equation of motion. For example, a nonlifting slender body of revolution can be simulated by a lineal source distribution or a lifting wing of zero thickness by a doublet sheet.

The study of a three-dimensional singularity distribution utilizing Eq. (2) can be simplified by allowing the control surface to recede indefinitely far from the object. Under this condition, it is possible to reduce the spatial representation to a series of one-dimensional distributions. The basis for this reduction for the first term is the finding by Hayes[30] that the velocity potential and the gradients of interest induced by an elementary solution of Eq. (1) along a trace on the distance control surface (PP' of Fig. 2) are invariant to a finite translation along the surface of a hyperboloid emanating from the trace and passing through the singularity. As the apex of the hyperboloid is a great distance away, the aforementioned movement is along a surface which is locally plane; it will be henceforth referred to as an "oblique plane." Since this solution satisfies a linear differential equation, all singular solutions which lie on the surface of the same hyperboloid may be grouped to form a single equivalent point singularity whose strength is equal to the algebraic sum of the individual strengths.

This finding provides the basic technique for reducing a general spatial distribution of singularities to a series of equivalent lineal distributions. This is accomplished by surveying the three-dimensional distribution longitudinally at a series of fixed cylindrical angles θ. At each angle, the survey produces an equivalent lineal distribution by systematically cutting the spatial distribution at a series of longitudinal stations along its length. At each cut, the group of intercepted singularities is collapsed along the "oblique plane" to form one of the equivalent point solutions comprising the lineal distribution.

Hayes'[30,31] far field expression for the wave resistance of a general system of volume, lift, and side force elements is

$$D_W = -\frac{\rho}{8\pi^2} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h'(\epsilon_1,\theta) h'(\epsilon_2,\theta) \times \ln|\epsilon_1 - \epsilon_2| d\epsilon_1 d\epsilon_2 d\theta \quad (3)$$

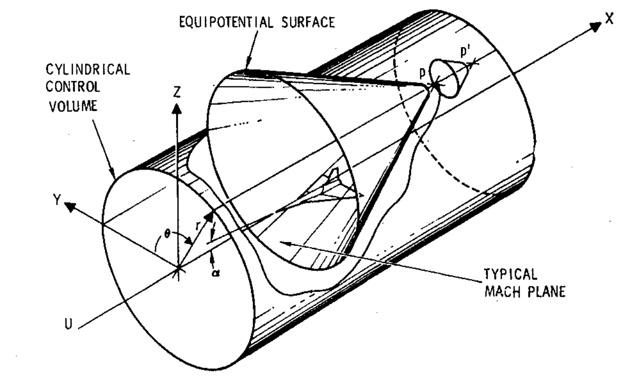

Fig. 2 Distant control surface geometry.

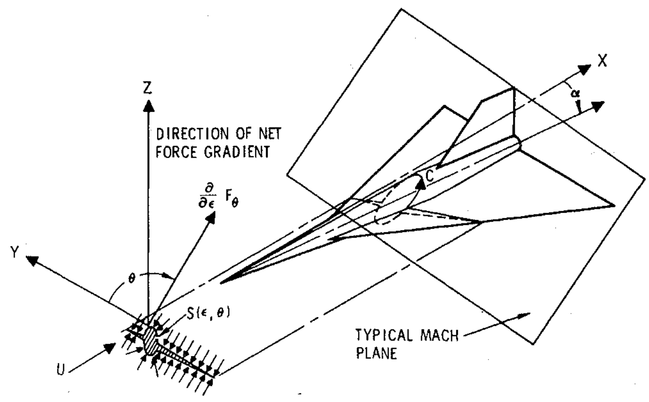

Fig. 3 Areas and forces pertinent to the evaluation of wave drag from the far field point of view.

where $h(\epsilon,\theta) = f(\epsilon,\theta) - g_z(\epsilon,\theta) \sin\theta - g_y(\epsilon,\theta) \cos\theta$ is the equivalent lineal singularity strength at the cylindrical angle θ; $f(\epsilon,\theta)$ = equivalent source strength per unit length; $(\rho U/\beta)g_z(\epsilon,\theta)$ = equivalent lifting element strength per unit length; $(\rho U/\beta)g_y(\epsilon,\theta)$ = equivalent side force strength per unit length.

The individual singularity strengths are related to the object under consideration by the requirement of flow tangency at the solid boundary. Lomax[32] derived the following expressions between the equivalent singularity strengths and a slender lifting object.

$$\begin{aligned} f(\epsilon,\theta) &= U(\partial/\partial\epsilon)S(\epsilon,\theta) \\ g_z(\epsilon,\theta) &= \frac{\beta}{2} U \int_C C_p dy \\ g_y(\epsilon,\theta) &= \frac{\beta}{2} U \int_C C_p dz \end{aligned} \quad (4)$$

where (see Fig. 3) $S(\epsilon,\theta) = y - z$ projection of the obliquely cut cross-sectional area; C = contour around the surface in the oblique cut.

The expression for vortex drag (second term of Eq. (2)] is also simplified by allowing the control volume to recede indefinitely far from the object under consideration since for this condition $\phi_x \rightarrow 0$ and thus,

$$D_V = \frac{\rho}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\phi_y{}^2 + \phi_z{}^2) dy dz$$

This equation may be transformed for a planar wake to

$$D_V = \frac{-\rho}{4\pi} \int_{-b/2}^{b/2} \int_{-b/2}^{b/2} \frac{d\Gamma(y)}{dy} \times \frac{d\Gamma(y_1)}{dy_1} \ln|y - y_1| dy dy_1 \quad (5)$$

where $b/2$ is the lateral extent of and Γ the circulation distribution in the wake. This result is the same as that obtained for induced drag in subsonic flow.

Volume

For this case, the second term of Eq. (2) is zero. Further, if the oblique loading (due to thickness) is small compared to the longitudinal gradient of cross-sectional area, eq. (3) reduces to the supersonic area rule form first proposed by Jones.[24]

$$D_W = \frac{-\rho U^2}{8\pi^2} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S''(\epsilon_1,\theta) S''(\epsilon_2,\theta) \times \ln|\epsilon_1 - \epsilon_2| d\epsilon_1 d\epsilon_2 d\theta \quad (6)$$

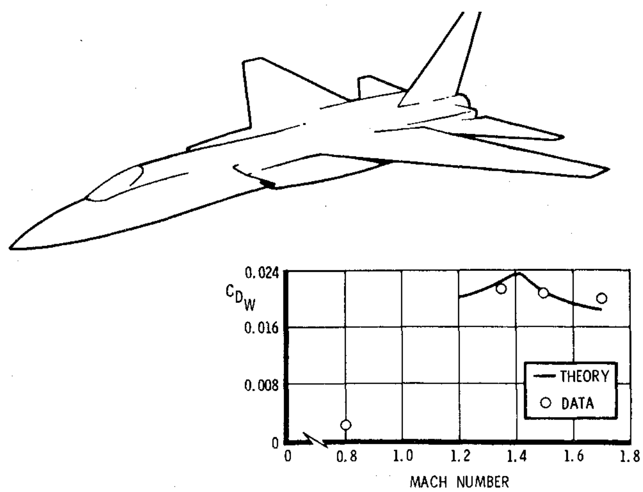

Fig. 4a Comparison of A3J theoretical and scale model wave drag due to volume.

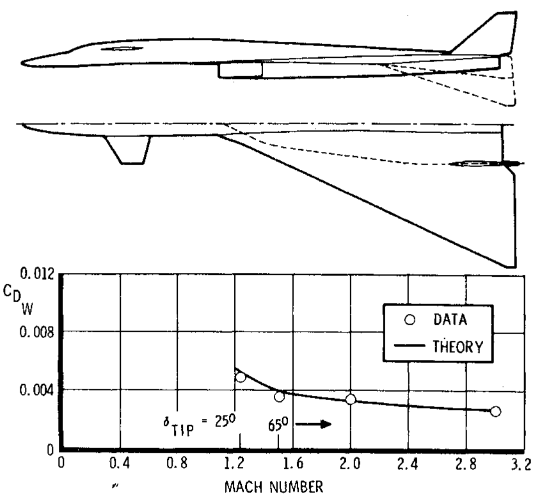

Fig. 4b Comparison of XB-70 theoretical and scale model wave drag due to volume.

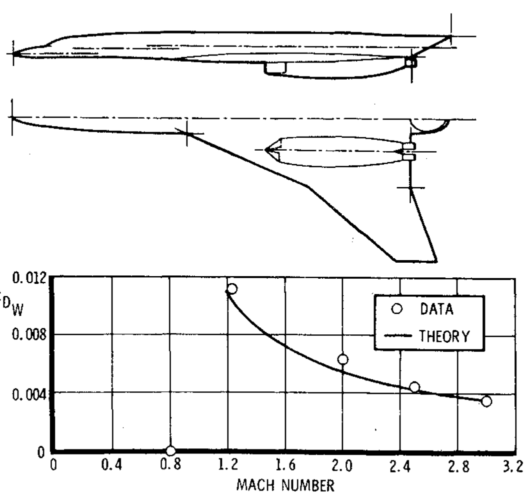

Fig. 4c. Comparison of transport theoretical and scale model wave drag due to volume.

Oswatitsch and Keune's equivalence rule[33] for general slender bodies and Whitcomb's transonic area rule[34] are special cases of this result for $M \rightarrow 1$ from previous. The supersonic area rule is in relatively widespread use[35,36] and is capable of providing a systematic theoretical analysis of complex three-dimensional distributions of volume for the subclass of objects satisfying the aforementioned approximation.

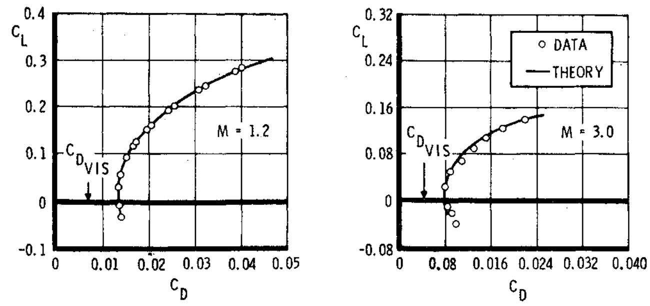

Fig. 5 Comparison of XB-70 theoretical and scale model drag characteristics.

A comparison between prediction utilizing Eq. (6) and measurement for wind-tunnel models of three contemporary aircraft concepts is presented in Fig. 4. The experimental wave drag was obtained by subtracting estimated turbulent skin friction and camber increments (if applicable) from the balance measurement. Numerous equally successful comparisons have previously been reported.[36–38]

From a design standpoint, the area rule greatly aids in the development of arangements having high volumetric efficiency through its ability to assess the interference (favorable or unfavorable) between components. Great emphasis is placed on distributing aircraft thickness to produce smooth high fineness ratio equivalent bodies at all radial angles.

Lift and Volume

The use of a distant point of view for the design of systems having both lift and volume has been relatively limited. This is due at least in part to the lack of a simple means (as compared to the previous volume problem) of obtaining an adequate definition of the loading strength required to evaluate the integrals of Eqs. (4) and (5). The techniques available for this purpose at the present time are lifting-surface theory and the wing-body analysis of Woodward.

Thus, a distant point of view for the lifting case must utilize near field techniques to obtain a solution. Although such an approach initially appears to be inconsistent and could be accomplished just as well by staying in the near field, the basic reason for using asymptotic techniques is to 1) incorporate favorable lift-volume interference concepts into aircraft design and 2) assess lift-volume interference not reflected in the theoretical techniques used to estimate the surface pressure.

From a design standpoint, low total system wave drag is obtained when the combined load and volume produce smooth high fineness ratio equivalent bodies

$$S_e(\epsilon,\theta) = S(\epsilon,\theta) - \frac{\beta}{2}\int_0^{\epsilon}\int_C C_p(\epsilon,\theta)[dy \sin\theta + dz \cos\theta]d\epsilon$$

at all radial angles.[39,40] The vortex drag relative to the lower-bound result for a plane wake of fixed span $D_V|_{\mathrm{MIN}} = 2L^2/\pi\rho U^2 b^2$ provides a direct measure of the lateral load efficiency. A comparison of the magnitude of the wave and vortex resistance in conjunction with the lower bound criteria gives the designer a means of determining whether modifications should be concentrated (and to what degree) on longitudinal lift/volume redistribution, span load shape, or both. A simple example will be given to illustrate this point. Consider a subsonic leading edge flat plate delta wing of zero thickness. It is known that the span load for this case is elliptical; hence, the vortex drag is minimum. Design modifications to reduce the wave resistance must concentrate on a longitudinal redistribution of loading. It is well known that this can be accomplished by warping the plate out of plane.

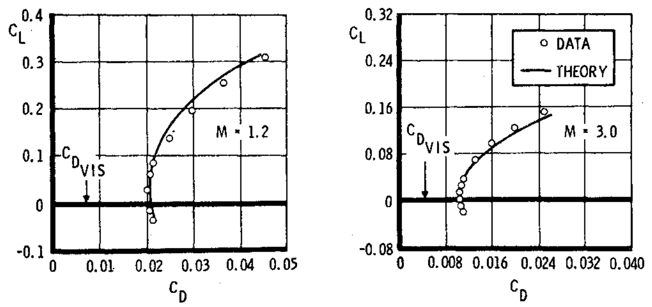

Fig. 6 Comparison of transport theoretical and scale model drag characteristics.

If changes in effective area shape are indicated, it is important to note that substantial volume redistribution (such as fuselage contouring, wing positioning, etc.) can often be made with relatively minor change in oblique load shape. This is quite helpful as low shock wave drag design considering lift-volume interference can be accomplished by modifying the loading and thickness independent of one another.

Comparisons of wind-tunnel measurements and predictions for lift-volume resistance based on a distant point of view will now be given. For each case, lifting-surface theory was utilized to obtain an estimate of the wing surface pressure due to lift. In the far field analysis, leading edge thrust is assumed to be fully effective. Since in practice this is seldom realized due to the action of compressibility and viscosity, it will be omitted even though such results are theoretically incomplete. This will be accomplished by requiring that the near and far field analysis for the zero thickness wing yield the same resistance. That is $D = D_W + D_V + D_{LE}$ where

$$D_{LE} = -T_{LE} = \rho U \int_{-b/2}^{b/2} \int_0^{c(y)} \phi_x \frac{\partial z}{\partial x}\, dxdy - (D_W + D_V)_{\text{zero thick wing}}$$

A comparison of prediction and scale model test results for two different concepts of a supersonic cruise vehicle is presented in Figs. 5 and 6. The composition of the drag for the transport wing body is presented in Fig. 7.

Less prediction success has been found for arrangements in which the fuselage was an appreciable fraction of the wing span. For such cases, it is necessary to account for the effect of the body on the wing load shape. Techniques such as that developed by Woodward may be used for this purpose.

Problem Areas

Although substantial progress has been made in the application of the linearized form of potential theory to the design of supersonic aircraft, several prolems requiring further attention remain.

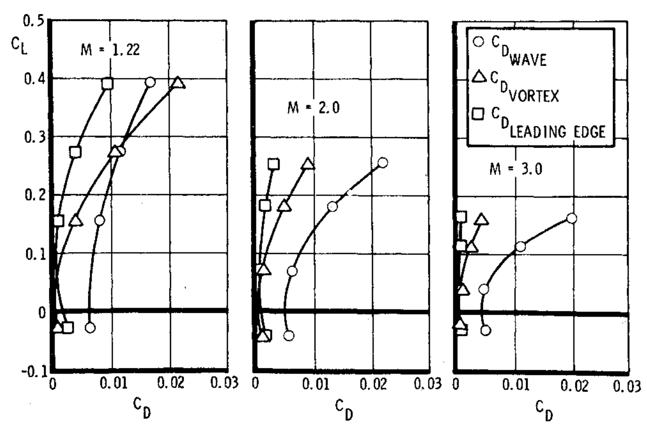

Fig. 7 Transport wing-body pressure drag composition.

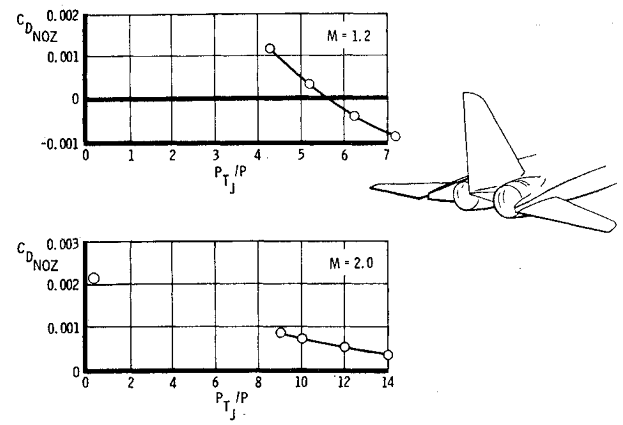

Fig. 8 Effect of a propulsive jet on nozzle drag at supersonic speeds, $A_J/A_T = 1.3$.

It is not intended here to present an exhaustive discussion of all possible difficulties which can be encountered but rather to reemphasize certain problems which, although certainly not new, remain unresolved.

One of these concerns afterbody design and drag evaluation. Propulsive jet flows interact with the external slipstream for underexpanded (jet static wall pressure at the exit is greater than local ambient) conditions and often cause substantial changes in the aerodynamic forces. Such interference generally occurs on upstream boattail surfaces and on aircraft components in proximity to and downstream of the jet exit. An experimental example of these interactions is presented in Figs. 8 and 9.

The interference of the jet with adjacent surfaces can be examined (at least in principle) using potential theory by treatment of the jet exhaust as a lineal source distribution whose strength is proportional to the volume divergence of the exhaust flow. That is, the external flow is assumed to interact with the propulsive jet as though it were a solid. Such an approach requires an estimate of the free jet gasdynamic boundary in a nonuniform back pressure environment. This problem to date has not been solved.

In supersonic flow, the interaction between a propulsive jet and upstream boattail surfaces can only exist because of the presence of fluid viscosity and hence cannot be explained in terms of potential theory concepts. This problem is apparently further complicated by not being amenable to a classical boundary-layer analysis since the equations of motion are parabolic (i.e., do not depend on downstream effects) and the pressure gradient normal to the wall is not small.

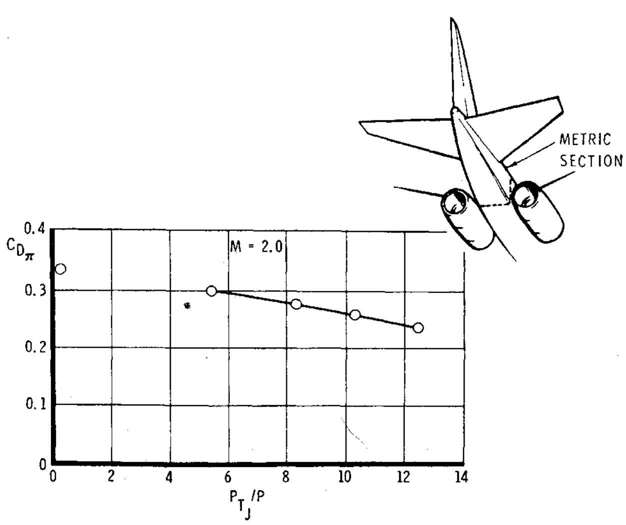

Fig. 9 Effect of a propulsive jet on fuselage—empennage drag.

A second problem concerns the theoretical analysis of lifting surfaces having subsonic edges. Numerical solutions are accomplished by subdividing the surface into a large number of rectangular or rhombic panels over which the upwash (source formulation) or pressure (vortex formulation) is assumed to be constant. Unless special care is taken to account for the singular behavior of panels near a subsonic leading edge, large fluctuations in the pressure distribution commonly result. Although this has not proved to be too bothersome in the evaluation of aerodynamic forces, the proper interpretation of the ability of a camber design to avoid premature separation is seriously compromised. That such consideration is necessary is clearly demonstrated by the study of Ref. 41.

Although numerical lifting-surface theory development is now capable of routinely evaluating the longitudinal aerodynamic characteristics of thin wings having "arbitrary" planform, the solutions are invariably restricted to supersonic trailing edge conditions. Published theoretical analysis of either analytic or numerical results removing this constraint is rare.

III. Conclusions

The numerical application of linearized potential theory to the external aerodynamic development of high-performance supersonic aircraft provides a highly useful design technique. The far field point of view in particular, with its emphasis on smooth high fineness ratio combined lift-volume distributions for low shock wave resistance and elliptical lateral load shape for minimum vortex drag, establishes clearly understandable low drag design goals and suggests specific courses of action to realize them.

Since the theory is a first-order inviscid approximation to the real flow, it is essential to continually verify experimentally that the predicated performance levels are in fact being realized and to what degree. Of equal importance is the definition of situations for which the theory is inadequate. A specific example of the latter is provided by the change in afterbody drag due to power.

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A General Theory of Aircraft Response to Three-Dimensional Turbulence

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An improved mathematical description of the dynamic response of an aircraft to three-dimensional isotropic turbulence is developed by establishing a unique correspondence between the spatial symmetry properties of the ambient field of turbulence and those of the aircraft, thereby yielding a more elegant and compact formulation offering significant advantages over the direct multiple input approach: 1) a 36-fold reduction in the size of the quadratic form that characterizes the response power spectral densities; 2) complete analytical separation between symmetric and antisymmetric response; 3) insight into the nature and origin of responses arising from interaction between longitudinal, lateral, and vertical gust velocity input components; 4) greater compatibility with conventional dynamic response computational procedures. The formulation is extended to include transfer functions, cross transfer functions, and coherence functions, such as are typically derived from dynamic response flight tests in which a gust reduction system is employed.

I. Introduction

THE accelerated development of large, subsonic aircraft and the availability of digital computers of increasing speed and capacity, has stimulated renewed interest in the problem of representing the full three-dimensional spatial dependence of the gust environment in dynamic response calculations. A straightforward multiple input treatment of the aircraft, as exemplified by Lin[1] and Fuller,[2] restricts physical insight and incurs a 36-fold redundancy in the mathematical description of the gust environment, which accordingly reduces computational efficiency. The formulation presented here avoids this redundancy and yields a comprehensive theory of gust response by employing two-point gust velocity input configurations to resolve the turbulence velocity field into two independent subfields, one symmetric and the other antisymmetric with respect to the aircraft geometry. Each subfield, by inducing structural responses of corresponding symmetry only, serves to maintain the conventional analytical separation between symmetric and antisymmetric response. Relevant properties of the turbulence field are derived by systematically introducing the assumptions of homogeneity, stationarity, isotropy, and Taylor's hypothesis. The following development thus may provide a point of departure for the investigation of cases involving modification of these assumptions.

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II. Decomposition of the Turbulence Field

Let $\mathbf{u}(\mathbf{r},t)$ represent the gust velocity at point $\mathbf{r}$ and time t, where $\mathbf{r}$ has coordinates (x,y,z) relative to an inertial frame. Reflection of a velocity vector about its local u_1,u_3 plane, and reflection of a position or separation vector through the x,z plane of the inertial frame will be indicated by affixing a prime. Thus, $\mathbf{u}' = (u_1,-u_2,u_3)$, and $\mathbf{r}'$ represents the point $(x,-y,z)$, as shown in Fig. 1. The relations between the components of the primed and unprimed vectors may be conveniently expressed as follows:

$$v'_i = (-1)^{i-1}v_i \tag{1}$$

where $\mathbf{v}$ is any vector.

Like Cartesian components of the gust velocity, impinging upon aerodynamic surface panels at bilaterally symmetric locations on a moving aircraft, eventually will be related to symmetric and antisymmetric two-point velocity configurations, which are denoted by $\mathbf{u}^+(\mathbf{r},t)$ and $\mathbf{u}^-(\mathbf{r},t)$, respectively, and are defined by

$$\mathbf{u}^{\pm}(\mathbf{r},t) = [\mathbf{u}(\mathbf{r},t) \pm \mathbf{u}'(\mathbf{r}',t)]/2 \tag{2}$$

so that

$$\begin{aligned} \mathbf{u}(\mathbf{r},t) &= \mathbf{u}^+(\mathbf{r},t) + \mathbf{u}^-(\mathbf{r},t) \\ \mathbf{u}'(\mathbf{r}',t) &= \mathbf{u}^+(\mathbf{r},t) - \mathbf{u}^-(\mathbf{r},t) \end{aligned} \tag{3}$$

Consider the ensemble averages of all possible products between velocity components $u^{\pm}_i(\mathbf{r}_m,t)$ and $u^{\pm}_j(\mathbf{r}_n,t+\tau)$ measured at points $\mathbf{r}_m$ and $\mathbf{r}_n$, and at times t and $t+\tau$, respectively. If the velocity field is homogeneous and stationary, then the ergodic hypothesis may be invoked to replace ensem-